# Lawrence Berkeley Laboratory
## UNIVERSITY OF CALIFORNIA

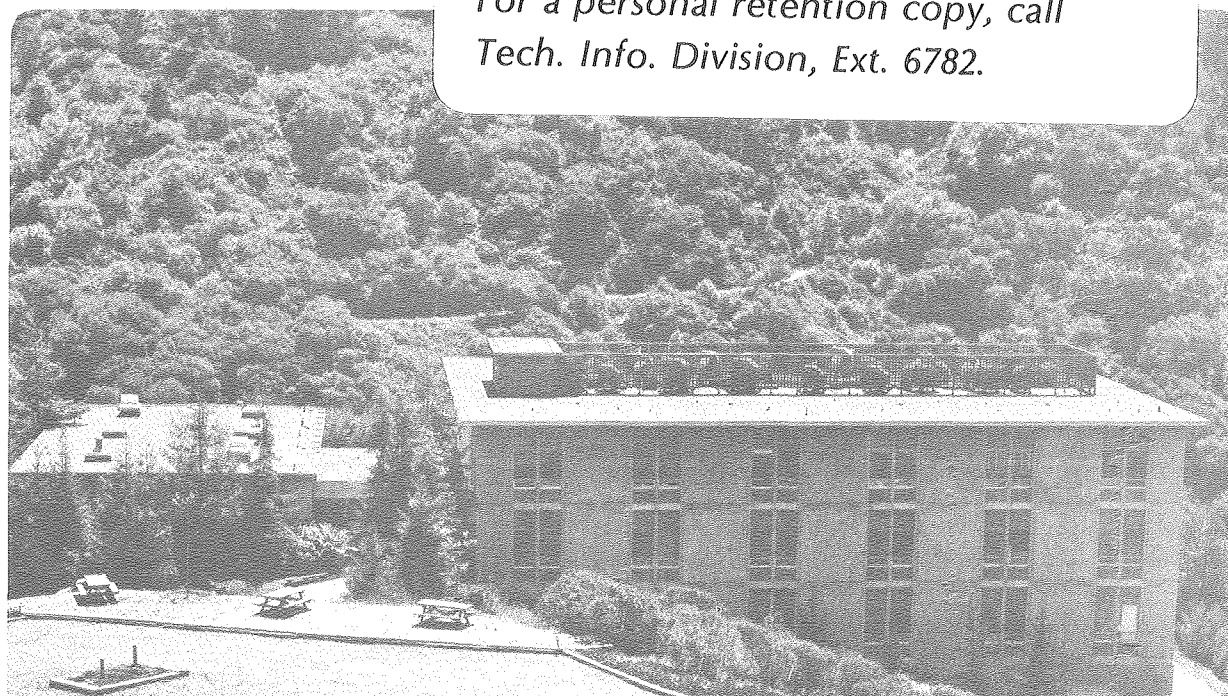## Materials & Molecular Research Division

OBSERVATION OF ZERO POINT FLUCTUATIONS IN A
RESISTIVELY SHUNTED JOSEPHSON TUNNEL JUNCTION

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## DISCLAIMER

OBSERVATION  OF ZERO POINT FLUCTUATIONS IN A

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Roger H. Koch, D. J. Van Harlingen and John Clarke

Department of Physics, University of California, and
Materials and Molecular Research Division, Lawrence
Berkeley Laboratory, Berkeley, California 94720

## Abstract

We have measured the spectral density of the voltage noise in current-biased resistively shunted Josephson junctions in which quantum corrections are expected to be important.  The I-V characteristics show self-resonances arising from the capacitance of the junction and the inductance of the shunt. When these resonances are taken into account, the measured noise is consistent with a model in which a large fraction of the noise originates in zero point fluctuations in the shunt resistor.

In a recent letter, Koch et al.[1] considered the effects of quantum corrections on the voltage noise in a current-biased resistively shunted Josephson junction. For measurement frequencies much less than the Josephson frequency and for a heavily overdamped junction they predicted a spectral density for the voltage noise

$$S_v(0) = R_D^2[(4k_BT/R) + (2eV/R)(I_0/I)^2 coth(eV/k_BT)]. \qquad (1)$$

Here, $I_0$, $C$, and $R$ are the critical current, capacitance, and shunt resistance of the junction, $I$ and $V$ are the current and voltage, and $R_D$ is the dynamic resistance. Equation (1) is based on the assumption that the noise arises from equilibrium noise currents in the shunt resistor with a spectral density $(2h\nu/R)$ coth $(h\nu/2k_BT)$ at frequency $\nu$. The first term on the right hand side of Eq. (1) represents noise from the resistor at the measurement frequency, while the second term arises from noise mixed down from frequencies near the Josephson frequency. In the limit $eV >> k_BT$, the second term represents zero point fluctuations; provided, in addition, $2eV(I_0/I)^2 > 4k_BT$, the measured noise will be dominated by the mixed-down zero point fluctuations. These limits require a junction with[1] $\kappa \equiv eI_0R/k_BT >> 1$. In this Letter we present experimental evidence for the observation of the zero point term. The measurement is complicated by the presence of resonances arising from the capacitance of the junction and the inductance of the shunt. However, the additional non-linearity introduced by these resonances has the fortunate effect of increasing the magnitude of the mixed-down noise; when the shunt inductance and junction capacitance are included in our model, the predicted noise is in good agreement with our experimental results.

We fabricated the junctions (inset, Fig. 1) by first depositing a 10 $\mu$m-wide Cu(3 wt. %Al) shunt 40 to 100 nm thick, followed by a 250 nm-thick Pb(20 wt. %In) film patterned into a 10 $\mu$m-wide cross-strip using a photo-lithographic lift-off procedure. The junction was defined by opening a 2.5 $\mu$m diameter window in a SiO insulating layer, while a second window set the shunt length at 5 $\mu$m. After patterning the resist for the upper electrode, the exposed metal surfaces were cleaned by rf sputter-etching in Ar, and the $In_2O_3$ oxide was grown thermally in $O_2$. Finally, the 400 nm-thick Pb counter electrode was deposited. This Pb film also formed a ground-plane for the resistive shunt, reducing its inductance to about 0.2pH. The junction capacitance was estimated to be 0.7pF, while the critical current ranged from 0.1 to 15 mA (0.2 to $30 \times 10^4 A$ $cm^{-2}$), depending on the oxidation parameters. The shunt resistance, typically 0.1$\Omega$, was chosen to be much less than the sub-gap quasiparticle tunneling resistance.

The low frequency voltage noise of each junction was amplified with a cooled 30 or 100 kHz LC-circuit coupled to a room-temperature low-noise pre-amplifier. The noise was mixed down to frequencies below 100 Hz, and the power spectrum of the low-frequency noise was measured with a computer. The junction current was supplied by batteries, with cold filters in the cryostat.

In each measurement, we determined the stray resistance in the tank circuit by measuring the Q with zero bias current in the junction; in the same way we determined the dynamic resistance at each current bias. The spectral density of the noise from the stray resistance (if significant) and the measured spectral densities of the preamplifier voltage and current noises were subtracted

from the total measured noise to give the junction noise. The system noise temperature was about 2.5K, while the uncertainty in the preamplifier noise and tank circuit gain contributed a mean error of $\pm 1 \times 10^{-24} V^2 Hz^{-1}$ for a typical junction. The system calibration was checked using the noise from known resistors at 300K, 4.2K, and 1.4K; the overall accuracy was $\pm 5\%$.

As a test of our measurement system and of the effectiveness of the shielding, we first measured junctions with $\kappa \ll 1$ in which Eq. (1) reduces to the Likharev-Semenov[2] (LS) result where quantum corrections are negligible:

$$S_V(0) = (4k_B T R_D^2/R) \ (1 + I_0^2/2I^2). \tag{2}$$

Figure 1 compares the measured noise with the predictions of Eq. (2), where we have used $R_D = R(1 - I_0^2/I^2)^{-\frac{1}{2}}$, R being the measured resistance at very high bias currents. The agreement above the noise-rounded region[3] is excellent.

We next studied junctions with $\kappa > 1$, and discovered the resonant structure illustrated in Fig. 2. In certain limits, the presence of the shunt inductance, $L_s$, can profoundly affect both the I-V characteristic and the voltage noise. The equivalent circuit is shown inset in Fig. 2. From our computer simulations, we find that the I-V characteristic will show substantial resonant structure similar to that shown in Fig. 2 when $\beta_L = 2\pi L_s I_0/\Phi_0 \gtrsim 1$ and the approximate Q of the series LCR circuit $(\beta_L/\beta_c)^{1/2} \gg 1$, where $\beta_c = 2\pi I_0 R^2 C/\Phi_0$. The LCR circuit pulls the Josephson frequency slightly so that it becomes more closely a subharmonic of the LCR resonant frequency. Hence, as the current bias is increased, the voltage will be alternately increased and decreased as the Josephson frequency passes through each subharmonic frequency of the LCR resonance. The 1/n dependence of the dynamic resistance is shown clearly

in Fig. 2 (n is an integer). In addition to the modification of the I-V characteristic, the increased non-linearity of the phase evolution results
other hand
in the mixing down of noise from multiples of the Josephson frequency; on the/ in the model leading to Eq. (1), such contributions are negligible.

We have investigated in detail two junctions with $\kappa > 1$ that exhibited strong resonances. Figure 3(a) shows the spectral density of the noise measured on one junction at 1.4K. The magnitude of the spectral density scales approximately as $R_D^2$. By measuring the total noise at 100 kHz and 30 kHz, we found that this junction also exhibited 1/f voltage noise. We subtracted the spectral densities of the 1/f noise at 100 kHz (typically 20% of the total) and of the noise at the measurement frequency, $4k_BTR_D^2/R$, from the total noise to determine the mixed-down noise at 100 kHz (we assumed T=1.4K). The mixed-down noise is also shown in Fig. 3.

We have computed the noise expected from the circuit shown inset in Fig. 2 using the equations of motion

$$I = I_0\sin\delta + C\dot{V} + I_S,\qquad (3)$$

and

$$V = I_S R + \dot{I}_S L_S + V_N.\qquad (4)$$

Here $\delta$ is the phase difference across the junction, $I_S$ is the current through $L_S$ and R, and $V_N$ is the equilibrium noise voltage generated by R with spectral density

$$S_V(\nu) = 2h\nu R\,\coth(h\nu/2k_BT).\qquad (5)$$

We measured R by suppressing the critical current with a magnetic field and chose the values of $L_s$ and C to try to match the observed resonant structure. The values of $L_s$ and C were consistent with the values estimated from the geometry. Figure 3 shows the computed spectral density of the mixed-down noise. For comparison, we show also the computed mixed-down noise using a noise spectral density for the resistor without the zero point term,

$$S'_V(\nu) = 2h\nu R[\coth(h\nu/2k_BT)-1] = 4h\nu R/[\exp(h\nu/k_BT)-1]. \qquad (6)$$

The result using Eq. (6) clearly underestimates the measured noise throughout the range of measurement.

The agreement between the measured noise and the noise computed using Eq. (5) is quite good below about 120 μV, but at higher voltages the measured noise lies somewhat above the predicted value. Furthermore, the measured and predicted resonances are mis-aligned, indicating that our choice of $L_s$ and C were slightly incorrect. We believe the discrepancy in magnitude arises from self-heating of the junction. From measurements of the noise in junctions with identical geometries in the low-κ limit in which quantum effects and self-resonances are unimportant, we estimate that the self-heating is about 1.7K per microwatt of power dissipated. Thus, at 200 μV we expect the temperature of the shunt to be about 1.0K above the bath temperature, thereby increasing the noise. This increase is small for the mixed down term, but significant for the noise generated at the measurement frequency. To allow for this effect, we have added the additional Johnson noise at the measurement frequency due to the increased temperature of the shunt to the computed mixed-down noise. In addition, we have also shifted the computed resonant structure

to align it with the measured structure; we emphasize that the necessary values of $L_S$ and C remain consistent with our estimated values. The effect of these two corrections on the computed spectral density is shown in Fig. 3(b): The agreement with the measured spectral density is now rather good.

We believe these results provide strong evidence first, for the existence of a zero-point term in the power spectrum of the voltage noise of a resistor in thermal equilibrium, and second, that these fluctuations give rise to the limiting noise in a resistively shunted Josephson junction in the quantum limit. Furthermore, the generally good agreement between our results and the predictions of our model justify our use of a Langevin treatment to predict quantum noise effects in a current-biased Josephson junction in the overdamped limit. Finally, in accord with earlier observations[4,5], we find no evidence for a contribution to the measured noise arising from the shot noise of pairs tunneling through the junction. If we assume that such a term could be treated on the same footing as the noise in the shunt resistor, the upper limit on the spectral density of the pair shot noise would be about $0.05(4eI_0)$. We emphasize, however, that these observations do _not_ invalidate the theory of Stephen[6], which is applicable to a different situation.
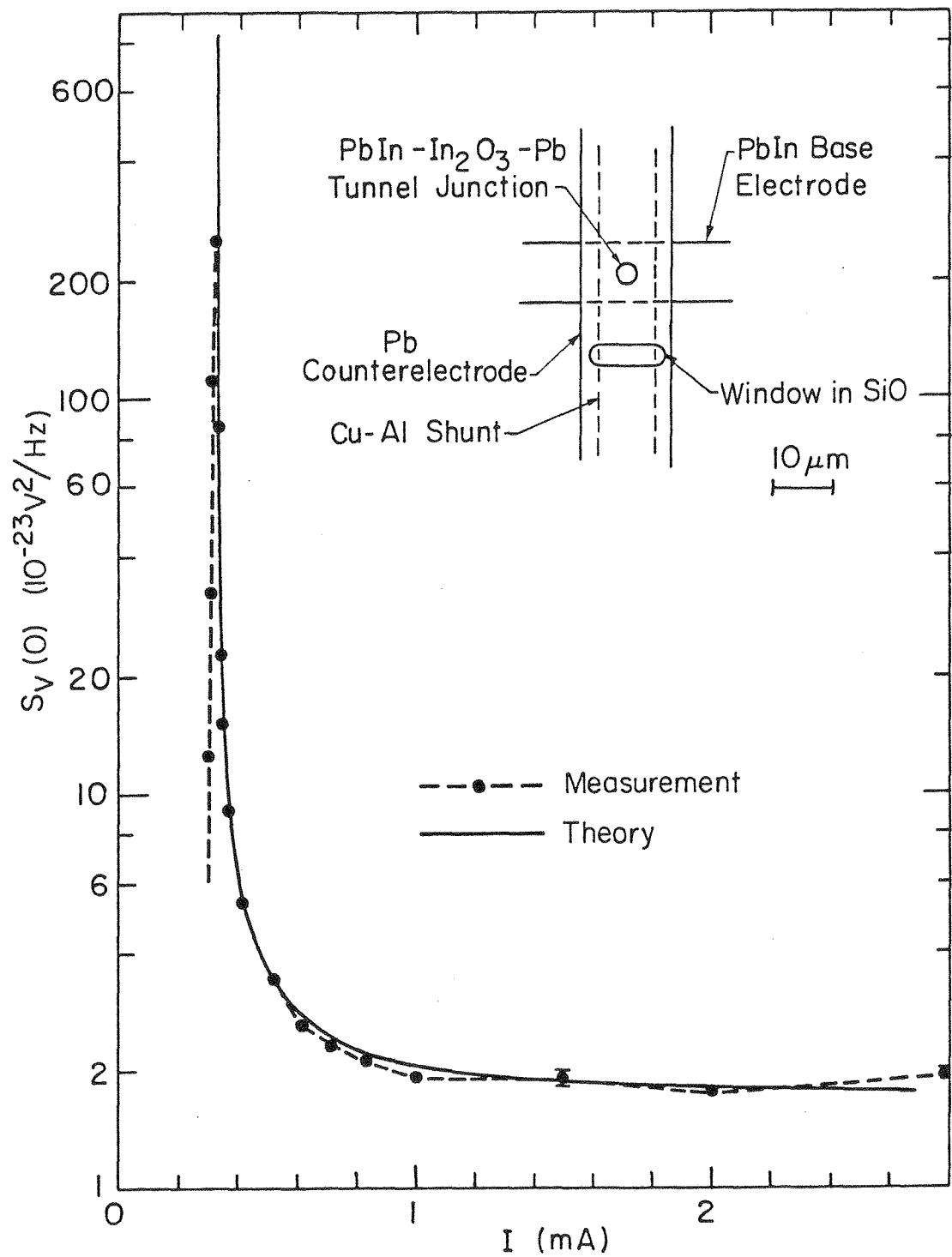
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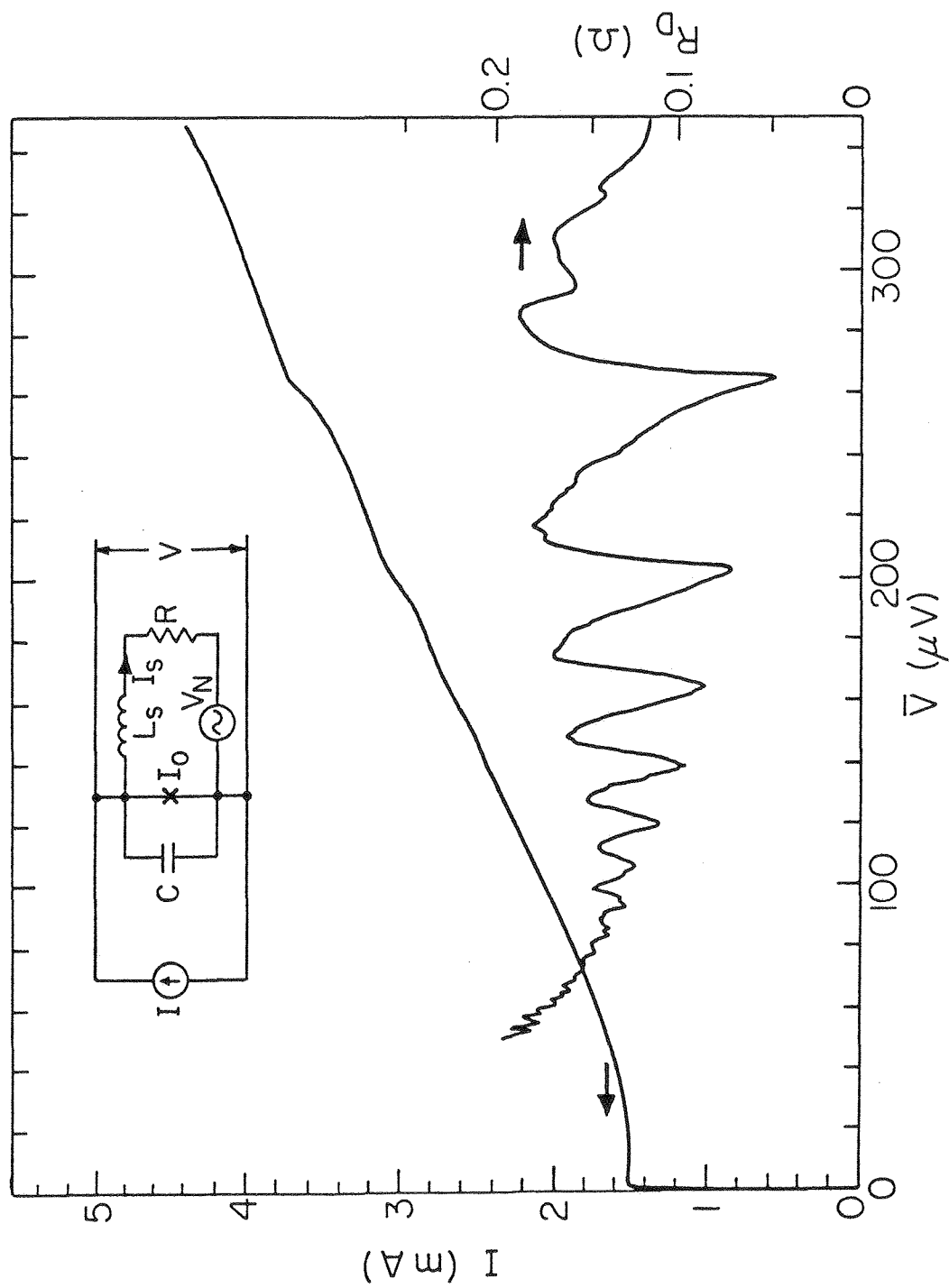
Figure Captions

Fig. 1    Spectral density of voltage noise vs. bias current for a junction

at $T = 4.2K$ with $I_0 = 0.32$ mA, $R = 0.075\Omega$, $\kappa = 0.066$, $\beta_L = 0.2$,

and $\beta_C = 0.004$.   Inset is junction configuration.

Fig. 2    I-V characteristics and dynamic resistance $R_D$ for a junction at

1.4K with $I_0 = 1.53$ mA, $R = 0.092\Omega$, $\kappa = 1.15$, $\beta_L = 1.0$, and

$\beta_C = 0.03$.   Inset is equivalent circuit of junction.

Fig. 3    Measured and computed spectral densities of the voltage noise for

the junction in Fig. 2.   In (a), the computed curves assume $T = 1.4K$,

while in (b), the additional Johnson noise at the measurement

frequency due to self-heating has been added, and the computed

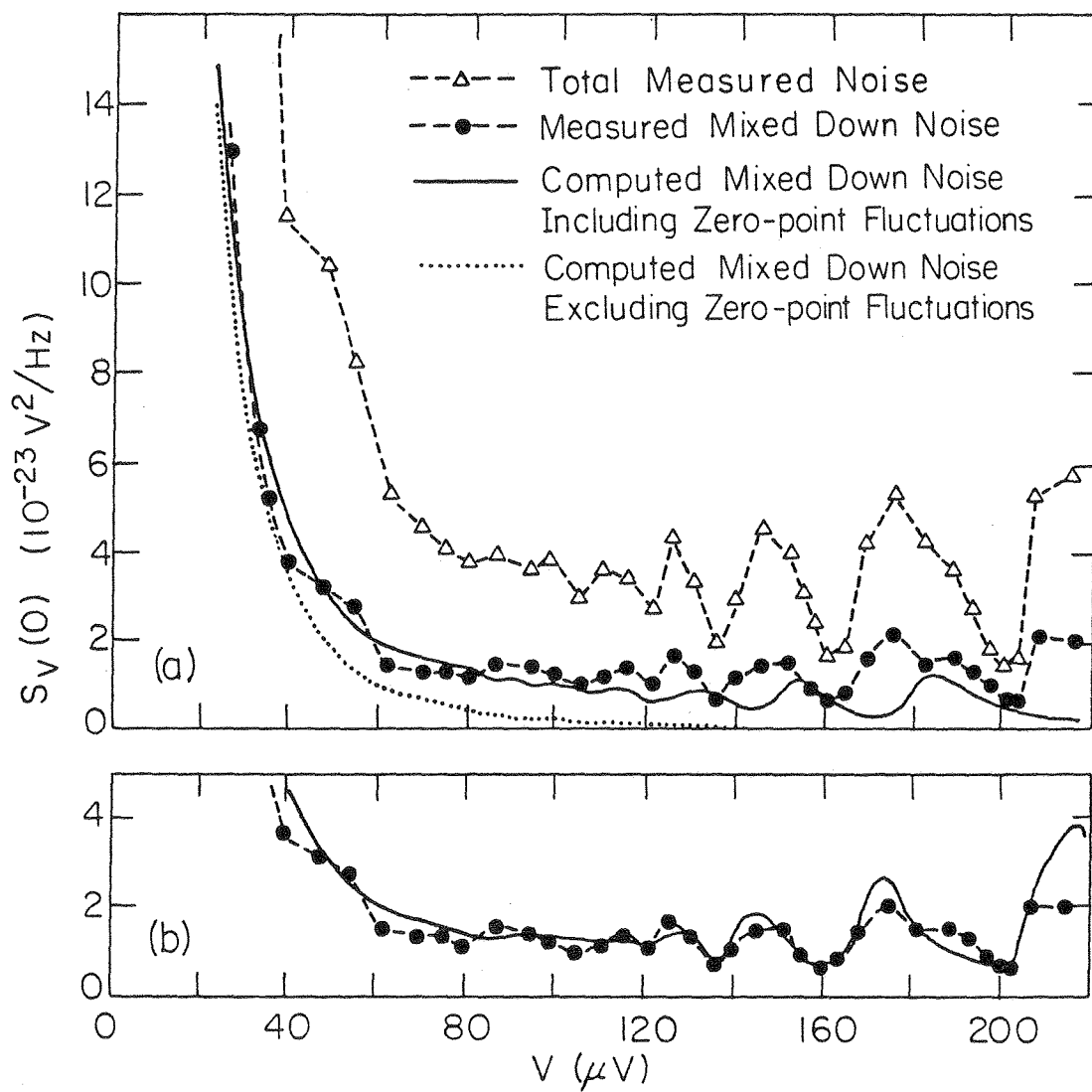resonances have been shifted to match the observed structure.

Fig. 1

Fig. 2

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Fig. 3

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